

# Search for higher flavor multiplets in partial-wave analyses

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**Abstract.** The possible existence of higher multi-quark flavor multiplets of baryons is investigated. We argue that the  $S$ -matrix should have poles with any quantum numbers, including those which are exotic. This argument provides a novel justification for the existence of hadrons with arbitrary exotic structure. Though it does not constitute a proof, there are still no theoretical arguments against exotics. We then consider  $KN$  and  $\pi N$  scattering. Conventional and modified partial-wave analyses provide several sets of candidates for correlated pairs  $(\Theta_1, \Delta)$ , each of which could label a related **27**-plet. Properties of the pairs (masses, mass orderings, spin-parity quantum numbers) do not quite correspond to the current theoretical expectations. Decay widths of the candidates are either wider or narrower than expected. Possible reasons for such disagreements are briefly discussed.

**PACS.** 11.55.Bq Analytic properties of S matrix – 11.80.Et Partial-wave analysis – 13.75.Gx Pion-baryon interactions – 13.75.Jz Kaon-baryon interactions

## 1 Introduction

Until very recently, it was commonly believed that all hadrons had only “minimal” quark structure. This meant that every meson could be considered as composed of only one quark-antiquark pair (or, perhaps, of gluons, without quarks at all), with every baryon composed of three quarks. This hypothesis implied a restriction on the possible quantum numbers of the hadrons (flavor numbers, in particular). However, nobody could suggest, in QCD or through other approaches, a mechanism to forbid hadrons containing additional quark-antiquark pairs and having exotic quantum numbers, incompatible with the “minimal” structure. This point was especially hot, since due to virtual gluon radiation and quark-antiquark pair production (*e.g.*, by vacuum polarization) every hadron has some probability to be seen in a configuration with (very) many quarks and gluons. Such configurations should emerge as short-time fluctuations of the initial state, even if it is “minimal”. Therefore, they may play the leading role in the hard (*i.e.*, short-time) processes. Indeed, such hard processes as deep-inelastic lepton scattering, Drell-Yan pair production, and so on, are known to be well described by structure functions, based on the presence in a hadron of both the constituent quarks (corresponding

to the “minimal” set) and the infinite number of “sea” quark-antiquark pairs.

The first attempts to theoretically understand the properties of multi-quark states (mainly, of mesons) were made in the framework of the MIT bag model [1]. Later, another method became popular for baryons. This approach is related to the chiral soliton model ( $\chi$ SM; see recent reviews [2,3] for more detailed description and references), and has allowed the clear prediction [4] of a new baryon state with unique properties. New experimental searches, stimulated by this prediction, have provided evidence for the first exotic baryon, the  $\Theta^+$  with mass  $M_{\Theta^+}$  about 1540 MeV and strangeness  $S = +1$ . The earliest positive data were obtained by the collaborations LEPS [5], DIANA [6], and CLAS [7]. Now, more than 10 publications support the existence of the  $\Theta^+$ , with decays to both  $K^+n$  and  $K_S p$ . Additional evidence for the  $\Theta^+$  (or some other exotic baryon(s) with  $S = +1$ ) has been demonstrated recently [8] in the properties of  $K^+$ -nuclear interactions.

Not all searches have yielded positive results. Some collaborations have not (yet) found the  $\Theta^+$  in their data. Of these negative results, some have been formally published (see, *e.g.*, refs. [9–19]), while others exist mainly as rumors, or as conference slides. Nevertheless, all of these cast doubt on the existence of the  $\Theta^+$ . Note that the negative results mainly correspond to higher energies than

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positive ones, and could be determined by different mechanisms. A new set of dedicated experiments, performed by several independent groups, are rather soon expected to provide more clear conclusions on the existence of this and other exotic hadrons.

The most decisive experimental check of the  $\Theta^+(1540)$  will come from good measurements of  $K^+n$  and/or  $K_Lp$  elastic scattering and/or charge exchange. Present data on these processes have insufficient quality and are only able to provide an upper bound for  $\Gamma_{\Theta^+}$  (not more than  $\sim 1$  MeV) [20–23]. In new precise measurements, with good resolution, the  $\Theta^+(1540)$  should manifest itself as a narrow peak, for which the height (up to the experimental resolution) may be calculated in an absolutely model-independent way. Such data would therefore allow one to either definitely confirm, or definitely exclude the  $\Theta^+(1540)$ .

A more detailed analysis of the existing data shows that, though the present non-observation data require exotic production to be small as compared to conventional hadrons, they do not entirely exclude the existence of the  $\Theta^+$  and/or its companions/analogues. For example, analysis of the BES data [9], presented in ref. [24], demonstrates some suppression of the  $\Theta$ -production. However, given the present experimental accuracy, this suppression is not severe, an essentially stronger suppression of the exotic production could still have a natural explanation. Similar conclusions apply also for other data sets (see, *e.g.*, ref. [13]). For this reason, we will assume the  $\Theta^+$  (as well as other multi-quark hadrons) to exist, and will discuss the consequences.

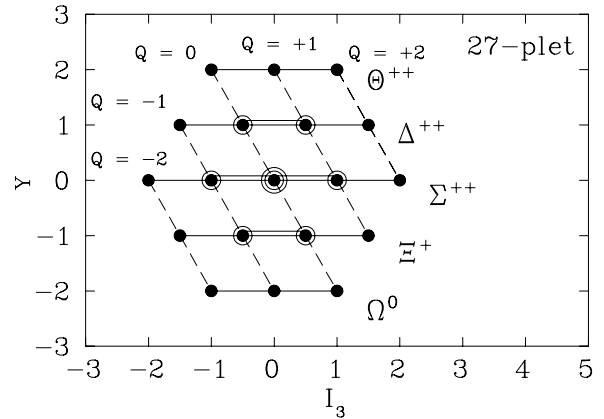
If all baryons were, indeed, classified as three-quark systems, then only a very limited set of flavor multiplets would be possible, corresponding to the product of three triplet representations  $\mathbf{3}$  of the flavor group  $SU(3)_F$ . Since

$$\mathbf{3} \times \mathbf{3} \times \mathbf{3} = \mathbf{1} + 2 \cdot \mathbf{8} + \mathbf{10}, \quad (1)$$

any baryon, in such a case, should either correspond to a flavor singlet  $\mathbf{1}$  (this is possible only for  $\Lambda$ -like baryons), or be a member of an octet  $\mathbf{8}$  or a decuplet  $\mathbf{10}$ .

A special role in this picture is played by non-strange baryons. Nucleon-like baryon cannot form a unitary singlet, nor can it be member of a decuplet. Therefore, in the conventional three-quark picture of baryons, a nucleon-like baryon can only be the member of an octet. It is natural, further, to suggest that every existing  $SU(3)_F$ -multiplet contains all possible states, even though the symmetry is violated and the mass degeneracy is broken. Then each  $N$ -like state should be accompanied by the whole set of associated octet states.

Thus, if only three-quark baryons existed, any  $N$ -like baryon could be used as a label for the corresponding flavor octet of baryons. In the same way, any  $\Delta$ -like baryon would be unambiguously related to the accompanying decuplet of baryons, and, therefore, could be a label for the corresponding set of states. In this short discussion, we have not accounted for a possible mixing between states of different multiplets. However, it is not essential for our present purpose of state counting: the mixing changes rela-



**Fig. 1.** Structure of a possible baryon  $\mathbf{27}$ -plet, for non-violated  $SU(3)_F$ . Hypercharge  $Y$  for baryons is  $S + 1$ . Full circles correspond to non-degenerate states. Each additional open circle corresponds to an additional state with the same values of  $Y$  and  $I_3$ , but with different value of  $I$ . Isotopic multiplets (constant values of the charge) are shown by solid (dashed) lines. Explicitly written for each value of  $Y$  are the baryons with the largest value of charge.

tions between different hadrons (relations of masses, of effective coupling constants and so on), but does not change the number of states.

If exotic hadrons do really exist, the situation becomes different from one considered above. Five-quark states may provide new flavor multiplets, according to the product relation

$$\mathbf{3} \times \mathbf{3} \times \mathbf{3} \times \mathbf{3} \times \bar{\mathbf{3}} = 3 \cdot \mathbf{1} + 8 \cdot \mathbf{8} + 4 \cdot \mathbf{10} + 2 \cdot \bar{\mathbf{10}} + 3 \cdot \mathbf{27} + \mathbf{35}. \quad (2)$$

If, for instance, the antidecuplet(s)  $\bar{\mathbf{10}}$  do(es) exist, some nucleon-like states may belong to antidecuplet(s), instead of octets. If even higher multiplets  $\mathbf{27}$  and  $\mathbf{35}$  exist, they should contain  $\Delta$ -like states not related with any decuplet. Moreover, the  $\mathbf{27}$ -plet, as well as many other (but not all) higher multiplets, contains both  $N$ -like and  $\Delta$ -like members (its multiplet diagram is shown in fig. 1; see also ref. [25] for some higher multiplet diagrams). The  $\mathbf{35}$ -plet, which may also be composed of 5 quarks, would have no  $N$ -like member (similar, in this respect, to the decuplet  $\mathbf{10}$ ). Instead, it would contain, in addition to the  $\Delta$ -like state, a new kind of non-strange states, having isospin  $5/2$ . Up to now, such states have not been explored at all.

Thus, existence of  $\bar{\mathbf{10}}$  and/or higher multiplets prevents  $N$ 's and  $\Delta$ 's from being good labels for flavor unitary multiplets. Instead, the higher multiplets have another characteristic member, a baryon with positive strangeness—the  $\Theta^+$ .

There is one crucial difference between  $\bar{\mathbf{10}}$  and even higher multiplets. Being a member of  $\bar{\mathbf{10}}$ ,  $\Theta^+$  is an isosinglet and has no positive-strangeness partners. In  $\mathbf{27}$ ,  $\mathbf{35}$ , and other higher flavor multiplets, the  $\Theta^+$  should be a member of a non-trivial isospin multiplet. Hence, it should be accompanied by isospin partners, also having  $S = +1$ . Each baryonic  $\mathbf{27}$ -plet contains isotriplet  $\Theta_1$ , with the subscript denoting its isospin  $I = 1$ . It consists of  $\Theta^0$ ,  $\Theta^+$ , and  $\Theta^{++}$ . Each  $\mathbf{35}$ -plet contains an isomultiplet  $\Theta_2$ , with

$I = 2$ , consisting of 5 members. In both cases, one of the isomultiplet members should be the double-charged exotic baryon  $\Theta^{++}$ . Note that no experiment gives evidence for the  $\Theta^{++}$  with mass near the  $\Theta^+(1540)$ , thus supporting its isospin  $I = 0$  and its expected antidecuplet nature. But if any  $\Theta^{++}$  exists, as a member of **27**, **35**, or even higher flavor multiplet, it should, in its turn, have the double-charged non-strange partner(s),  $\Delta^{++}$  with  $I = 3/2$  (for **27**, see fig. 1), or even two such states, corresponding to  $I = 3/2$  and  $I = 5/2$  (for **35**).

From this brief discussion, we see that the  $\Theta^{++}$  could be used as a evidence for multiplets higher than  $\overline{\mathbf{10}}$ . Note, however, that

$$\mathbf{8} \times \mathbf{8} = \mathbf{1} + 2 \cdot \mathbf{8} + \mathbf{10} + \overline{\mathbf{10}} + \mathbf{27}. \quad (3)$$

Since the “stable” (*i.e.*, having no strong decays) mesons and baryons (except the heavy  $\Omega^-$ -baryon) belong to flavor octets, we see that among multi-quark multiplets only members of  $\overline{\mathbf{10}}$  and/or **27** can decay to a pair of such “stable” hadrons or appear as a resonance in their scattering. For instance, the isosinglet  $\Theta^+$ , a member of  $\overline{\mathbf{10}}$ , can decay to  $KN$ . The  $\Theta^{++}$ , as seen in  $K^+p$  interaction, evidently cannot have  $I = 0$ , as should be for the  $S = +1$  member of  $\overline{\mathbf{10}}$ . But if isospin violation is negligible, it cannot also have  $I = 2$ , as would be necessary for a **35**-plet. This implies the impossibility to search for **35** or further higher flavor multiplets in 2-hadron processes (in particular, in meson-baryon elastic scattering). The same is true for processes of electro- or photoexcitation of the nucleon, where the photon appears as the member of an  $SU(3)_F$  octet, and for 2-hadron mass distributions in multi-hadron final states.

That is why, in what follows, we will discuss mainly **27**-plet, considering it as an example of higher flavor multiplets. A characteristic feature of the **27**-plet is the presence of both  $\Theta^{++}$  and  $\Delta$  members. They must be correlated, in the sense that their spin and parity should be the same; their masses may differ, since the  $SU(3)_F$  is violated, but we do not expect them to be widely separated.

Of course, the **27**-plet should also contain an  $N$ -like member. However, eq. (2) shows that the 5-quark system may reveal numerous  $N$ -like states, not related to **27**-plets (one for each  $\mathbf{8}$  and/or  $\overline{\mathbf{10}}$ ). Therefore, the pair correlation of  $\Theta^{++}$  with  $N$  (or triple correlation of  $\Theta^{++}$ ,  $\Delta$ ,  $N$ ) is less characteristic than the pair correlation of  $\Theta^{++}$  with  $\Delta$ . So, generally, we will not look for it here.

In the next section, we give a new argument for the existence of  $S$ -matrix poles with any quantum numbers, including exotic ones. We also discuss which poles can be related to physical resonances.

In section 3, we examine existing data from  $KN$  and  $\pi N$  scattering, with  $I = 1$  and  $3/2$  respectively, to search for possible  $\Theta^{++}$  and  $\Delta$  candidates with the same values of  $J^P$  and with correlated masses. We will investigate the possibilities of both wide and narrow states.

The obtained results and their meaning are discussed in the concluding section 4.

## 2 Resonance spectroscopy and complex-energy poles

As is well known, stable particles are related to poles in energy for some scattering amplitudes. These poles appear at real energies below the lowest physical threshold of the corresponding scattering channel (for the relativistic description, it is more convenient to use not the energy itself, but the c.m. energy squared). By analogy, we will consider resonances (unstable bound states) as poles at complex values of the energy. In what follows, we will study resonances, in particular, in physical channels with exotic quantum numbers. Up to now, the mere existence of such states has been doubted. As a result, we begin with discussion of this issue.

Here we encounter two sets of questions. Some of these may be called theoretical: How many (if any) complex poles may an amplitude have? Which poles may be considered as corresponding to resonances? Other questions are more phenomenological. They are related to the search for amplitude poles starting from experimental data. These sets of questions are physically similar, though they look and are treated differently. Let us consider them in more detail.

### 2.1 Theoretical questions

It is the standard assumption that the strong-interaction amplitudes have definite analytical properties. For the amplitudes of 2-particle-to-2-particle processes they provide dispersion relations in both the energy and momentum transfers. Most such relations have never been formally proved on the basis of general axioms from Quantum Field Theory (a rare exception is the energy dispersion relation for pion-nucleon forward scattering). Quantum Chromodynamics, which is believed to underlie strong interactions, also cannot be used today to prove the dispersion relations for hadronic amplitudes, since even the transition from quarks and gluons to hadrons has not yet been traced without any additional assumptions and in a model-independent way. Nevertheless, such relations are widely used in the phenomenological treatment of strong interactions. For instance, analytical properties are assumed to extract meson-meson elastic amplitudes from data on processes involving meson-nucleon transformation into 2-meson-nucleon. Various dispersion relations are used now as input in modern partial-wave analyses (PWA; see, *e.g.*, the latest  $\pi N$  PWA [26]). They are also applied to extract such strong interaction parameters, as meson-nucleon coupling constant(s) and the so called  $\sigma$ -term. Up to now, the application of dispersion relations has not induced any inconsistencies. Therefore, it seems reasonable to investigate consequences of the dispersion relations in more detail, in particular, with respect to the problem of exotic states.

The assumption of the momentum-transfer dispersion relations for strong-interaction amplitudes of 2-particle-to-2-particle hadronic processes implies the possibility of analytical continuation for the corresponding partial-wave amplitudes to complex values of angular momentum  $j$

(the Gribov-Froissart formula [27], see also the monograph [28]).

A strong-interaction amplitude may have poles in the complex energy plane, each with definite  $j$  (which physically takes only discrete integer or half-integer values). Instead of such energy-plane poles, one can consider Regge poles, in the complex  $j$ -plane, their positions and residues being dependent on energy. Positions of the two kinds of poles are connected by a relation of the form

$$F(E, j) = 0,$$

and have one-to-one correspondence. Therefore, we may apply the above questions to the Regge poles, instead of the energy-plane poles. Such an approach allows us to answer the first question, about the number of poles.

As was shown by Gribov and Pomeranchuk [29], when the energy  $E$  tends to a value of  $E_{\text{th}}$ , the (elastic or inelastic) threshold of two spinless particles, the Regge poles accumulate to the point

$$j = l = -1/2. \quad (4)$$

Here  $j$  is the total angular momentum, while  $l$  is the orbital angular momentum of the two interacting particles (of course, they generally differ, but coincide for the system of two spinless particles). The movement of accumulating Regge poles near threshold is described by the trajectories

$$l(E) \approx -\frac{1}{2} + \frac{i\pi n}{\ln(R\sqrt{-k^2})} + \mathcal{O}(\ln^{-2}(R\sqrt{-k^2})), \quad (5)$$

with  $R$  and  $k$  being the effective interaction radius and relative c.m. momentum. The number  $n$  takes any positive and negative integer values,  $n = \pm 1, \pm 2, \dots$ . Just below the threshold (at  $k^2 < 0$ ) the first two terms of eq. (5) describe infinitely many pairs of reggeons, which are complex conjugate to each other and tend to the accumulation point at  $k^2 \rightarrow 0$ . Note that relativistic amplitudes always have many thresholds, for production of two- or many-particle intermediate states. More detailed investigation of the correction terms in eq. (5) shows that the simple pair-wise complex conjugation of the poles is exactly true near (below) the lowest threshold, but is approximate near the higher thresholds, being violated by small correction terms. This complex conjugation ordering becomes destroyed also just above the threshold (even the lowest one), where  $k^2 > 0$  and  $\ln(-k^2)$  is complex.

These results are quantitatively not general: for the threshold of two particles with spins  $\sigma_1$  and  $\sigma_2$  there appear several accumulation points [30], the rightmost one is at

$$j = -1/2 + \sigma_1 + \sigma_2, \quad (6)$$

instead of (4). The reason is simple: the accumulation points still correspond to  $l = -1/2$ , but particle spins provide several possible  $j$ -values for any fixed  $l$ -value, and *vice versa*. Correspondingly, the structure of the accumulations is still described by trajectories (5), with the shifted limiting points in the  $j$ -plane. The case of

multi-particle thresholds has never been really explored (though hypothesized by Gribov and Pomeranchuk [29]).

The qualitative nature of the results is general enough. The accumulations are directly related to the threshold behavior  $\sim (k_i)^{l_i}(k_f)^{l_f}$ , characteristic for the strong-interaction amplitudes without massless exchange contributions (here  $k_i$  and  $k_f$  are the relative c.m. momenta,  $l_i$  and  $l_f$  are the orbital momenta, and the subscripts  $i$  and  $f$  correspond to the initial and final states).

The Gribov-Pomeranchuk (GP) accumulation phenomenon is not a specific property of relativistic amplitudes; it emerges also in the case of non-relativistic potential scattering with the finite-range  $R$  of interaction, where the scattering amplitude has the threshold behavior  $\sim (kR)^{2l}$  (the accumulation was explicitly demonstrated [31], in particular, for the Yukawa potential  $V(r) = g \exp[-\mu r]/r$ , having the effective interaction radius of order  $1/\mu$ ).

Essential for us now is the infinite number of the accumulating Regge poles [29], which implies that the total number of the Regge poles is certainly infinite as well. For the partial-wave amplitude of two-particle interaction with the fixed angular momentum, this corresponds to the infinite number of poles in the energy plane (all Riemann sheets). For the non-relativistic case of the Yukawa potential, in the limit  $\mu \rightarrow 0$  (transforming the Yukawa potential into the Coulomb one, with an infinite radius of interaction), the infinite number of energy-plane poles is seen as the infinite number of Coulomb radial excitations. In such a limit, the GP accumulation of Regge poles near the threshold takes the form of accumulation of Coulomb bound states to the threshold [31] (note that the double limiting transition  $\mu \rightarrow 0$ ,  $k \rightarrow 0$  is not equivalent here to the similar, but reversed limit  $k \rightarrow 0$ ,  $\mu \rightarrow 0$ ).

We emphasize that the above arguments have not assumed specific quantum numbers in the scattering channel. Therefore, their consequences should be equally applicable (or non-applicable) to both bosonic and fermionic hadron poles, with any flavor quantum numbers (exotic or non-exotic).

Thus, if we study  $2 \rightarrow 2$  strong-interaction amplitudes, we should admit the existence of (an infinite number of) complex-energy poles with any exotic quantum numbers, both mesonic and baryonic. Alternatively, one could assume that analytical properties of amplitudes having exotic quantum numbers for at least one of physical channels ( $s$ -,  $t$ -, or  $u$ -channels) are essentially different from those of totally non-exotic amplitudes. However, we consider the latter case to be unnatural. Thus, we obtain one more argument for the existence of exotics, absent in the previous publications.

Several earlier arguments for exotics were collected and briefly discussed in ref. [24]. One of them also uses reggeons and, thus, might look similar to the present result. However, they are essentially different. That older evidence for exotics [32] was based on ‘‘duality’’ of resonances and reggeons, which was understood as equivalence between the sum of resonances in the direct channel ( $s$ -channel) and the sum of reggeons in the exchange

channel ( $t$ - or  $u$ -channel), see ref. [28]. The arising evidence for exotics can be avoided, *e.g.*, by assuming a “conspiracy” of resonances or reggeons. Our reasoning is different, using reggeons in the direct channel and their one-to-one correspondence with familiar energy poles in the same channel. The result in this case cannot be so easily circumvented.

Note that all suggested arguments for exotics, either theoretical or phenomenological, give evidence, but cannot prove existence. We emphasize here, however, that no firm theoretical arguments have been presented to forbid exotic hadrons. Until either exotics are observed, or their absence is understood, one cannot be convinced that the present picture of strong interactions is complete and self-consistent.

Now we encounter another theoretical question: whether every complex-energy pole should be considered as possibly related with a resonance. The problem is that, in terms of complex angular momenta, the Regge poles participating in the GP accumulations are clearly separated from the physical points, which are, for the  $j$ -plane, only the integer (or half-integer) non-negative points. Thus, these poles cannot provide physically meaningful (bound or resonance) states, at least near the corresponding threshold.

It might be that the total set of Regge poles is split into two (or more?) different subsets: one related, say, with the GP accumulations, another with the bound and/or resonance states. Then there could be an infinite number of poles of the former type, while only few (if any) poles of the latter type. If so, there could appear no bound or resonance states with exotic quantum numbers, though there are infinitely many poles somewhere on Riemann sheets of the energy plane.

Such a problem was also investigated for the Schrödinger equation with the Yukawa potential [33]. Results obtained there show no basic difference between various Regge poles: all Regge trajectories appear to be different branches of the same multi-valued analytical function. Formally, this fact is related to non-trivial analytical properties of the Regge trajectories as functions of the energy: their singularities come not only from physical thresholds, but also from coincidence of two (or more) reggeons [33], where those reggeons can be interchanged.

The non-relativistic scattering off the Yukawa potential seems to provide a good test-ground for analytical properties of the  $2 \rightarrow 2$  relativistic scattering amplitudes. Thus, it is natural to think that all poles in the energy plane at fixed angular momentum for relativistic amplitudes also have a common nature. Then the notion of a resonance becomes rather conventional: *the pole to be considered as related to a resonance state should be placed not too far from the physical region*, to produce noticeable enhancement in the physical amplitudes. This means, in particular, that the resonance width should not be too large. Quite conventionally, today we may take, say,  $\Gamma^{tot} < 500$  MeV. Such a boundary may be increased in future, when both experimental precision and theoretical understanding are improved.

Though we have shown, under rather standard assumptions, that the  $S$ -matrix has an infinite number of energy poles for any exotic quantum numbers, this is only a necessary condition for exotic physical states to emerge. A sufficient condition would be to prove that some of those energy poles can be placed near the physical region. For such goal, however, we need to know more detailed dynamics.

## 2.2 Phenomenological questions

Similar problems, though apparently different, arise also in phenomenological approaches. Consider, *e.g.*, the case when one extracts an amplitude from experimental data, and then continues it into the complex energy plane to search for poles. Evidently, experimental data are given at discrete energies (we neglect experimental uncertainties for the moment). It is well known, however, that analytical continuation from a discrete set of points is ambiguous. Therefore, the position of a pole found from experimental data, and even its existence, formally speaking, may be ambiguous as well. Quantitatively, such ambiguity is the smaller, the nearer to experimental points is the complex energy value, reached in the continuation. Thus, the problem of searching for poles becomes most ambiguous for poles far from the physical region. We see again that only states with not very large widths are physically reliable.

Near the physical region, ambiguity of the continued amplitude should be quantitatively small. Nevertheless, a pole may still be ambiguous, if it has a sufficiently small residue, which corresponds to a small total and/or partial decay width.

Up to now, we have considered the possible pole uncertainties as a mathematical problem. However, experimental errors and “technical” methods used in extracting amplitudes may result in additional ambiguities for the states of either very large or very small width. For instance, as we discussed in ref. [34], the standard procedures for PWA may miss narrow resonances with, say,  $\Gamma^{tot} < 20$ –30 MeV. Note that the conventional PWA for elastic scattering may miss also a resonance with  $\Gamma^{tot} > 30$  MeV, if it has small elastic partial width  $\Gamma^{el}$ , providing a small elastic branching ratio of, say, less than 5%.

Now, given an understanding of the possibility, and even necessity, of complex-energy poles with various flavor quantum numbers, including exotic ones, we are ready to discuss the present status of such poles on the basis of existing experimental data.

## 3 Resonance states in partial-wave analyses

As explained in the introduction, we will search for  $S$ -matrix poles with  $S = +1$ ,  $I = 1$  on one side, and with  $S = 0$ ,  $I = 3/2$  on the other. Formally, this purpose could be achieved by studying only scattering of  $K^+p$  and  $\pi^+p$ , respectively. However, wider sets of experimental data may be involved by studying all available charge

combinations of  $KN$  and  $\pi N$  scattering, with later separation of isospin states. In this way one can use the latest published PWAs for  $KN$  [35] and  $\pi N$  [26] amplitudes correspondingly. We will discuss their application separately for wide and narrow states.

### 3.1 Wide states

Energy-dependent PWA allows one to analytically continue the partial-wave amplitudes into the complex-energy plane and to search there for poles of the amplitudes. For  $KN$  scattering with isospin  $I = 1$ , the latest PWA [35], up to  $W = 2650$  MeV, reveals two poles having not too large imaginary parts:  $(M, \Gamma^{tot}/2) = (1811, 118)$  MeV for the  $P_{13}$ -wave, and  $(M, \Gamma^{tot}/2) = (2074, 253)$  MeV for the  $D_{15}$ -wave. We will consider these as candidates for the states  $\Theta_1$  (the subscript denotes the isospin value  $I = 1$ ) with  $J^P = 3/2^+$  and  $J^P = 5/2^-$  correspondingly.

As explained in the introduction, the  $\Theta_1$ , being a member of **27**-plet, is to be accompanied by a  $\Delta$ -like partner. For the two above states, such companions should have the same values of  $J^P$ . Therefore, they are expected to be seen as poles in the  $\pi N$  partial-wave amplitudes  $P_{33}$  and  $D_{35}$ .

Indeed, the latest PWA for the  $\pi N$  scattering [26], up to  $W = 2260$  MeV, contains the pole in the  $D_{35}$  amplitude with  $(M, \Gamma^{tot}/2) = (1966, 182)$  MeV. Quite reasonably, it may be the  $\Delta$ -like companion for the  $\Theta_1$  of  $(M, \Gamma^{tot}/2) = (2074, 253)$  MeV with  $J^P = 5/2^-$ .

What can be said about the  $P_{33}$  amplitude? The Review of Particle Properties contains two candidates, the  $P_{33}(1600)$  and  $P_{33}(1920)$  with widths in the 200–400 MeV range [36]. These are based on earlier PWAs, including previous VPI results [37] (where only  $P_{33}(1600)$  was seen). Note that evidence for  $P_{33}$  states above the  $P_{33}(1232)$  is weak in elastic  $\pi N$  scattering, possibly because of small,  $< 20\%$ , elastic branching ratios. The latest published analysis of  $\pi N$  data [26] does not give reliable candidates in the appropriate energy range.

One could assign either of the above states to be the  $\Delta$ -like companion for the  $\Theta_1$  of  $(M, \Gamma^{tot}/2) = (1811, 118)$  MeV with  $J^P = 3/2^+$  (though both candidates have masses largely separated from the  $P_{13}$  state found in  $KN$  scattering). However, we consider  $P_{33}(1920)$  to be less reliable (smaller elastic branching ratio, the mass is nearer to the upper end of the energy ranges used in PWAs). The position of the pole for  $\Delta(1600)$ , and even its existence, looks rather uncertain as well, possibly due to influence of nearby thresholds,  $\pi\Delta$  and  $\rho N$ . This state and its properties need confirmation that might come from detailed studies of inelastic processes (*e.g.*,  $\pi N$  and/or  $\gamma N$  production of the  $\pi\pi N$  final state). At the present moment, we will tentatively use  $\Delta(1600)$  with PDG-values of its parameters [36].

Summarizing the situation for wide resonances, we have two pairs of candidates for **27**-plet members:

$$J^P = \frac{3^+}{2}, \quad (M_{\Theta_1}, \Gamma_{\Theta_1}^{tot}) = (1811, 236) \text{ MeV}, \\ (M_{\Delta}, \Gamma_{\Delta}^{tot}) = (1600, 300) \text{ MeV}; \quad (7)$$

and

$$J^P = \frac{5^-}{2}, \quad (M_{\Theta_1}, \Gamma_{\Theta_1}^{tot}) = (2074, 506) \text{ MeV}, \\ (M_{\Delta}, \Gamma_{\Delta}^{tot}) = (1966, 364) \text{ MeV}. \quad (8)$$

Note that the number of wide  $\Delta$ -states in the  $\pi N$  analysis [26] is larger than the number of wide  $\Theta_1$ -states in the  $KN$  analysis [35]. While the pairs  $(\Theta_1, \Delta)$  are candidates for **27**-plets, the excess  $\Delta$ 's may correspond to more familiar decuplets, without any  $\Theta_1$ -companions.

We can recall here that every **27**-plet should contain also one  $N$ -like state. Baryon tables [36], indeed, demonstrate possible wide candidates, even two for each of pairs (7) and (8).

For the pair (7), with  $J^P = 3/2^+$ , one candidate is the  $P_{13}$  state  $N(1720)$  (with 4-star status [36]). It is present also in the new solution FA02 [26], with the pole parameters  $(M_{\text{pole}}, \Gamma_{\text{pole}}^{tot}/2) = (1655, 139)$  MeV or the Breit-Wigner parameters  $(M_{\text{BW}}, \Gamma_{\text{BW}}^{tot}/2) = (1750, 128)$  MeV. The properties of  $N(1720)$  seem to agree with  $\chi\text{SM}$ -calculations [3, 38] for the **27**-plet with  $J^P = 3/2^+$ . Another candidate could be the  $P_{13}$  state  $N(1900)$ , if it exists (it has  $\Gamma^{tot} \approx 500$  MeV and only 2 stars [36]; the solution FA02 [26] does not contain this state).

For the pair (8), with  $J^P = \frac{5^-}{2}$ , one candidate is the  $D_{15}$  state  $N(1675)$ . It also has 4-star PDG-status [36] and is also present in the solution FA02, with the pole parameters  $(M_{\text{pole}}, \Gamma_{\text{pole}}^{tot}/2) = (1659, 73)$  MeV and the Breit-Wigner parameters  $(M_{\text{BW}}, \Gamma_{\text{BW}}^{tot}/2) = (1676, 76)$  MeV [26]. The second candidate could be one more  $D_{15}$  state  $N(2200)$ , with  $\Gamma^{tot}$  of 300–400 MeV and 2 stars [36]. It does not appear in the FA02 [26] (probably as this is near the upper boundary of the analysis).

### 3.2 Narrow states

As has been explained above (for more detailed discussions see in refs. [34, 39]), any PWA by itself tends to miss narrow resonances. That is why we suggested [34] to modify the PWA procedure by explicitly assuming the existence of a narrow resonance and comparing the quality of fits with and without this addition (more details of the method and related formulas are in ref. [39]). Such an approach was used initially to place restrictions on light resonances in pion-nucleon scattering [34].

This method was applied also to studies of  $\Theta^+(1540)$  [21], giving strict limitations on the  $\Theta^+$  quantum numbers and, especially, its width, confirmed by results of other approaches. This method was then used [39] to search  $\pi N$  scattering data for a narrow nucleon-like state assumed to be a member of the antidecuplet, accompanying the  $\Theta^+(1540)$ . The two discovered candidate mass regions have recently obtained preliminary experimental support from direct measurements by the STAR and GRAAL collaborations [40, 41]. Thus, the modified PWA looks to be a useful instrument in the search for narrow resonances.

Let us recall some features of this approach. To the canonical PWA procedure, we add the explicit assumption that there is a resonance with fixed parameters. We scan the assumed value of the resonance mass, and also values of the total width and (for the inelastic resonance) the elastic partial width (instead of the elastic partial width, one may scan the elastic branching ratio). Then the database is fitted with and without this resonance hypothesis. Evidently, our procedure introduces additional parameters and, at first sight, should always decrease  $\chi^2$ . However, this is not necessary true, since those additional parameters are given with a specific functional form (narrow resonance) that may be admissible for some values of parameters, but definitely not for others. We consider the resonance to be possible if  $\chi^2$  with the additional resonance is smaller than without it.

Our experience shows that this approach becomes inefficient for resonances with large total width  $\Gamma^{tot}$  (if it is larger than, say, 30 MeV). In this sense, our method appears to be complementary with the conventional PWA, which is sensitive just to resonances with larger values of  $\Gamma^{tot}$ . Moreover, for an inelastic resonance, our approach is mainly sensitive to the upper boundary for the elastic partial width  $\Gamma^{el}$ , but is not so sensitive to the particular value of  $\Gamma^{tot}$  (see discussion in ref. [39]).

Here, we apply the modified PWA to search for possible members of a **27**-plet, which could be seen as narrow resonances in kaon-nucleon and pion-nucleon scattering. For the  $KN$  scattering, with the threshold near 1440 MeV, we take the c.m. energy interval from 1500 MeV up to 1750 MeV, below the  $K\Delta$  threshold. For the  $\pi N$  scattering, we take the same interval. We investigate  $S$ ,  $P$ , and  $D$  partial-wave amplitudes, essentially involved in the PWAs of refs. [26, 35]. As a result, we find several candidates in each of the waves, for both  $KN$  and  $\pi N$  scattering. The elastic partial widths are restricted to very small values. For instance, the  $S_{31}$ -wave of  $\pi N$  interaction suggests three  $\Delta$ -like candidates having  $J^P = 1/2^-$  and  $(M, \Gamma^{el}) = (1570 \text{ MeV}, < 250 \text{ keV})$ ;  $(1630 \text{ MeV}, < 30 \text{ keV})$ ;  $(1740 \text{ MeV}, < 90 \text{ keV})$ . There are candidates with even smaller  $\Gamma^{el}$ , down to 10 keV, but we do not consider them here. In this respect, we recall once more that our approach, generally, cannot prove the existence of any narrow resonance. Our modification of an amplitude has a definite functional form (corresponding to a narrow resonance). However, it may appear successful even without any true resonance, *e.g.*, as an imitation of some other features of the amplitude, taken into account in the conventional (unmodified) PWA, but with insufficient accuracy (if considered at all, see the discussion in ref. [34]). Therefore, we consider our candidates only as evidence that the corresponding values of energies (masses) are worth more detailed experimental investigations.

After removing the most doubtful candidates, we have the set of possible narrow resonances summarized in table 1 for the  $S$ -wave ( $J^P = 1/2^-$ ), two  $P$ -waves ( $J^P = 1/2^+$  and  $3/2^+$ ), and two  $D$ -waves ( $J^P = 3/2^-$  and  $5/2^-$ ). It is interesting that we see equal numbers of candidates

**Table 1.** Candidates for narrow  $\Delta$ - and  $\Theta_1$ -like states in  $\pi N$  and  $KN$  scattering.

$J^P$	$(M_\Delta, \Gamma_\Delta^{el})$ (MeV, keV)	$(M_{\Theta_1}, \Gamma_{\Theta_1}^{el})$ (MeV, keV)
$1/2^-$	(1570, <250)	(1550, <80)
	(1630, <30)	(1640, <100)
	(1740, <90)	(1740, <60)
$1/2^+$	(1550, <400)	(1530, <100)
	(1680, <50)	(1660, <80)
	(1730, <30)	(1740, <100)
$3/2^+$	(1550, <100)	(1530, <80)
	(1660, <30)	(1650, <50)
	(1720, <70)	(1710, <30)
$3/2^-$	(1520, <50)	(1530, <150)
	(1570, <120)	(1570, <70)
	(1730, <60)	(1740, <80)
$5/2^-$	(1510, <50)	(1530, <70)
	(1570, <30)	(1570, <60)
	(1620, <15)	(1640, <100)
	(1700, <70)	(1680, <60)

for both  $\Delta$ - and  $\Theta_1$ -like narrow states, contrary to the case of wide resonances.

## 4 Discussion of the results

Let us first summarize the results of the preceding section. We do see several correlated pairs of possible resonances  $\Theta_1$  and  $\Delta$ , each having the same spin-parity and nearby masses, which may be considered in a natural way as members of **27**-plets. Note that such an interpretation could be spoiled if we saw more  $\Theta_1$  than  $\Delta$  candidates. However, we have not encountered this problem, though both the data sets and the analyses for  $KN$  and  $\pi N$  scattering are totally independent. Indeed, an excess of  $\Delta$  candidates does not prevent the **27**-plet hypothesis, since the excess  $\Delta$ 's may be related to other flavor multiplets, *e.g.*, to decuplets having no open exotics. For excess  $\Theta_1$ 's, no reasonable multiplet prescription would be possible.

Further, we see two classes of candidates. The first contains very broad states, with total widths of hundreds MeV. The second is restricted to very narrow widths. Here the elastic width  $\Gamma^{el}$  could be as small as 100 keV, or even tens of keV. These values coincide with the total widths  $\Gamma^{tot}$  of  $\Theta_1$ -like candidates having masses in the elastic region (below  $\sim 1570$  MeV). Other  $\Theta_1$ -like candidates, with higher masses, and all  $\Delta$ -like candidates, may have  $\Gamma^{tot} > \Gamma^{el}$ . However, we study the  $\Theta_1$ -masses below the  $\Delta K$  and  $NK^*$  thresholds, and expect the inelastic contribution to be moderate. Thus, though we cannot reliably extract the total widths of these narrow candidates, our procedure, in any case, suggests that they, most probably, should have  $\Gamma^{tot}$  small, not larger than low tens of MeV.

Let us compare our candidates with theoretical expectations. The  $\Theta^+$  and the whole antidecuplet with  $J^P = 1/2^+$  were predicted [4] on the basis of the chiral soliton model ( $\chi$ SM). Definitely, such an approach also predicts two **27**-plets, one with  $J^P = 3/2^+$  and one with  $J^P = 1/2^+$  (see *e.g.*, refs. [2, 3]; note that the case of  $J^P = 1/2^+$  has usually been discussed much more briefly). Instead, among our candidates there are even several correlated pairs  $(\Theta_1, \Delta)$  with  $J^P = 3/2^+$ , several pairs with  $J^P = 1/2^+$ , and also pairs with different combinations of spins and parities, not considered in  $\chi$ SM. All those pairs are expected to label the corresponding **27**-plets.

Properties of our candidates for **27**-plet members look different from the conventional  $\chi$ SM expectations. For instance, estimations in  $\chi$ SM [3] for the **27**-plet with  $J^P = 3/2^+$  have given the  $\Theta_1$  to be about 60 MeV heavier than the antidecuplet  $\Theta^+$ , and its  $\Delta$  companion about 50 MeV heavier than the  $\Theta_1$ . Similar predictions were also given in other papers, *e.g.*, in refs. [2, 38].

Instead, the  $\Theta_1$ -state in our wide candidate pair of eq. (7) is heavier than the  $\Theta^+$  by more than 250 MeV. The corresponding  $\Delta$ -state could be lighter or heavier than the  $\Theta_1$ -state by about a hundred MeV, depending on the assignment. For narrow pairs (in particular, for the pairs with  $J^P = 3/2^+$ ), as shown in table 1, the masses of the  $\Theta_1$  and  $\Delta$  companions are nearly the same.

Thus, none of our candidate pairs correspond to the expected  $(\Theta_1, \Delta)$ -mass ordering. Evidently, the  $\Theta_1$ -masses, with respect to the  $\Theta^+$ , are also different from expectations. Note that such conclusions would not change if we used other pair combinations of  $\Theta_1$ 's and  $\Delta$ 's having the same  $J^P$ -values.

Expectations for widths of the **27**-plet baryons have also been discussed in the literature, though mainly for explicitly exotic states. The width of the  $\Theta_1$  with  $J^P = 3/2^+$  was estimated in  $\chi$ SM to be between 37 and 66 MeV [3, 38]. Our candidates, again, do not demonstrate the expected values. They have widths either essentially wider (more than 200 MeV), or essentially narrower (less than 1 MeV). Width estimations for the  $\Delta$ -member of the same **27**-plet,  $\Gamma^{tot} \sim 100$  MeV and some tens of MeV for the partial width  $\Gamma^{el}$  of the  $\pi N$  decay [38], also do not correspond to the properties of our candidates.

The current literature suggests some other, alternative to  $\chi$ SM, approaches to describe members of higher baryon multiplets. For example, the QCD sum rules [42] predict  $\Theta_1$  (and even  $\Theta_2$ , with  $I = 2$ , belonging to **35**) to have  $J^P = 1/2^-, 3/2^-$  and masses nearly the same as the  $\Theta^+$ , though positive parities are not excluded. It appears, however, that the suggested alternative approaches similarly fail to describe our candidates.

Nevertheless, we do not insist that the  $\chi$ SM (or any other model) is incorrect for higher flavor multiplets. The problem is that the published calculations in the framework of a particular model always use some additional assumptions, not inherent in the model itself. For the  $\chi$ SM, states considered in the literature are rotational excitations of the soliton. As a result, all baryon states in the  $\chi$ SM publications have only positive parities, though

negative-parity baryon resonances are certainly known to exist in nature [36]. Excitations of other types, vibrational for example, are also possible, though methods for their study have not been elaborated. Note that the vibrational excitations might provide a more numerous set of states than the rotational ones. Such states could have opposite parities and essentially smaller widths as compared to the states with rotational excitations.

In any case, existence of two kinds of states, very wide and very narrow, would hint at a very interesting dynamics. Note that similar effects emerge even for the rotational excitations in the  $\chi$ SM. Indeed, the coupling of decuplet baryons to octet states is well described and provides rather large width of, say,  $\Delta(1232)$ . On the other hand, theoretical estimates of the antidecuplet-octet coupling reveal some suppression [4], though its origin has not yet been understood (experimentally, the suppression seems to be even stronger than expected). A similar situation might arise also for the higher flavor multiplets. An interesting fact in this respect is that among our narrow candidates there are no excess  $\Delta$ -like states, which could belong to decuplets. This might mean that only higher multi-quark multiplets can have suppressed hadronic widths.

In discussing masses and widths of the candidate states, one should take into account the influence of their mixing(s) with states having the same flavor and spin-parity quantum numbers, but belonging to other flavor multiplets. Such mixing may essentially shift the expected masses and decay properties of multi-quark baryons, as has been suggested and demonstrated, *e.g.*, in refs. [39, 43–45]. The large number of vibrationally excited states might be very important in this respect.

We should again emphasize that our results give evidence for candidate states, but cannot prove their existence. Therefore, direct experimental searches, with good precision and good mass resolution, become important in the final decision. As for the  $\Theta^{++}$ , several searches for this state in  $K^+p$  mass distributions have been published [46–51], all with negative results. This is quite natural, having in mind the widths of our candidates. Indeed, it would be difficult to separate from background a bump with the width of hundreds of MeV, which should correspond to our wide candidates for  $\Theta_1$  in pairs (7) and (8). On the other hand, the manifestation of a peak with a very small width,  $< 1$  MeV, especially with small production cross-section, would be essentially suppressed by the experimental resolution  $\sim 10$  MeV.

To conclude, we have argued that the  $S$ -matrix (and, thus, the corresponding amplitudes) should have poles with any quantum numbers, exotic or non-exotic. Thus, a necessary condition for physical exotic states to exist is satisfied, though we cannot prove that the poles reveal themselves indeed as exotic hadrons. Nevertheless, our result presents a new argument for such a possibility. When studying the  $KN$  and  $\pi N$  scatterings, we see in their PWAs the baryon candidates,  $\Theta_1$ 's and  $\Delta$ 's, for members of several multi-quark **27**-plets. Properties of the candidates (their quantum numbers, masses and widths) do not quite correspond to expectations published



in the literature. This might have a natural explanation, but the existence of our candidates should still be checked more reliably by direct measurements, with good accuracy and resolution.

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